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A Detailed Study of Deterministic Inventory Control Models with Numerical analysis

J. *Kiruthika* [1] and B. *Kanishka* [2]

Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India.

M.Sc., Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

ABSTRACT:

This paper presents a systematic mathematical study of Deterministic Inventory Control Models under the assumption of constant demand, known lead time, and fixed cost parameters. The study focuses on the formulation and optimization of classical models including the Basic Economic Order Quantity (EOQ) model, EOQ model with shortages, Production Inventory model, EOQ model with quantity discounts, and multi-item deterministic inventory models. For each model, total inventory cost functions are developed and optimized using differential calculus to obtain optimal order quantities and minimum costs. The analysis demonstrates how deterministic models provide clear decision rules for cost minimization and efficient inventory management under certainty conditions.

KEYWORDS:

Deterministic Inventory Model, Economic Order Quantity (EOQ), Inventory Optimization, Holding Cost, Ordering Cost, Shortage Model, Production Model, Quantity Discount Model, Multi-Item Inventory, Operations Research.

INTRODUCTION:

Inventory management is a fundamental area of Operations Research concerned with determining optimal policies for ordering and storing goods.

Maintaining excessive inventory increases holding cost, whereas insufficient inventory leads to shortages and loss of goodwill. Hence, the primary objective of inventory control is to minimize total inventory cost while satisfying demand.

Inventory models are broadly classified into deterministic and probabilistic models. In deterministic models, parameters such as demand rate, ordering cost, holding cost, and lead time are assumed to be known and constant. These assumptions allow the development of precise mathematical formulations and analytical solutions.

The Economic Order Quantity (EOQ) model forms the foundation of deterministic inventory theory. Over time, several extensions have been developed to address practical situations, including shortages, finite production rates, quantity discounts, and multi-item systems. This paper presents a detailed analytical study of these deterministic models, emphasizing cost function formulation and optimization techniques. The objective is to demonstrate how mathematical model contributes to efficient and economical inventory decision-making.

Meaning of Deterministic Model:

A deterministic model is a type of mathematical or analytical model in which all the inputs and parameters are known with certainty and produce a predictable, fixed outcome. There is no randomness or uncertainty in a deterministic model-given the same inputs, it will always produce the same results.

Key Points:

- Predictable results - outcomes can be calculated exactly.
- Known parameters - all variables like demand, cost, or time are certain.
- No randomness - unlike stochastic models. there is no probability or variation involved.

- Decision-making - helps in planning and optimization with precise calculations.

Example:

If a factory produces 50 units/day, the cost of production is $10/unit, and it operates 20 days/month, a deterministic model can exactly calculate the monthly production cost:50 x 10 x 20 = 10,000

In Inventory Management:

A deterministic inventory model assumes that demand, lead time, ordering cost, and holding cost are known and constant, allowing managers to calculate the optimal order quantity and reorder time without uncertainty.

Deterministic Inventory Model:

A Deterministic Inventory Model is an inventory management model in which all the key variables and parameters are assumed to be known with certainty and remain constant over time. These models are used to determine the most economical way to manage inventory by calculating the optimal order quantity and reorder schedule.

In deterministic models, there is no uncertainty or randomness involved in factors such as demand, lead time, ordering cost, or holding cost. As a result, the outcomes of these models are precise and predictable.

Main Features of Deterministic Inventory Models

1. Known Demand: The demand for inventory is fixed and does not fluctuate.
2. Fixed Lead Time: The time between placing an order and receiving it is constant.
3. Known Costs: Ordering cost, holding cost, and shortage cost (if allowed) are predetermined.

4. Predictable Results: Since all parameters are certain, optimal policies can be calculated exactly.
5. Mathematical Optimization: These models use mathematical formulas to minimize total inventory cost.

Objective of Deterministic Inventory Models:

- The main objective of deterministic inventory models is to:
- Determine the optimal order quantity (EOQ)
- Decide the best time to place an order
- Minimize total inventory cost, which includes:
  - Ordering cost
  - Holding (carrying) cost
  - Shortage cost (if applicable)

Types of Deterministic Inventory Models:

Common deterministic inventory models include:

- Economic Order Quantity (EOQ) Model – without shortages
- EOQ Model with Shortages
- Production Inventory Model (EPQ Model)
- EOQ Model with Quantity Discounts

Each of these models is designed for specific inventory situations depending on whether shortages are allowed, production is involved, or price discounts are available.

Importance of Deterministic Inventory Models:

Deterministic inventory models play a crucial role in inventory management and operations research. They provide a simple and systematic approach for making effective inventory decisions when demand and costs are known with certainty. The importance of deterministic models can be understood through the following points:

1. Cost Minimization

Deterministic models help organizations minimize total inventory cost by balancing:

- Ordering cost
- Holding (carrying) cost
- Shortage cost

By calculating the optimal order quantity, these models ensure that inventory is managed at the lowest possible cost.

2. Simple and Easy to Apply

Since all parameters are known and fixed, deterministic models are:

- Easy to understand
- Simple to implement
- Suitable for practical decision-making

They do not require complex probability calculations.

3. Helps in Effective Decision-Making

Deterministic models provide clear answers to important questions such as:

- How much inventory should be ordered?
- When should an order be placed?
- What will be the total inventory cost?

This supports managers in making quick and accurate decisions.

4. Better Inventory Planning

These models help organizations to:

- Maintain optimal stock levels
- Avoid overstocking

- Prevent stockouts
- Ensure smooth production and supply chain operations

5. Foundation for Advanced Models

Deterministic models form the basic foundation for more advanced inventory models such as:

- Stochastic (probabilistic) models
- Fuzzy inventory models
- Dynamic and multi-item models

Understanding deterministic models is essential before moving to complex real-world inventory systems.

6. Useful for Graphical and Numerical Analysis

Deterministic models allow:

- Exact mathematical calculations
- Clear graphical representation
- Sensitivity analysis of costs and demand

This makes them very useful for academic study as well as industrial applications.

MODEL I – Basic Economic Order Quantity (EOQ) Model:

Definition:

The Basic EOQ Model is a deterministic inventory model used to determine the optimal order quantity that minimizes total inventory cost when demand and costs are known and shortages are not allowed

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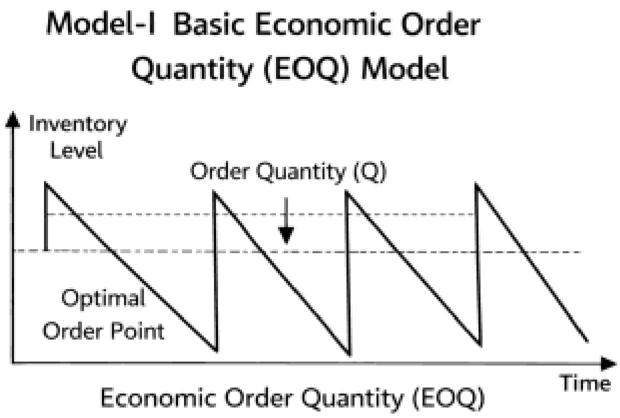

Concept:

This model is based on balancing two major costs:

- Ordering cost
- Holding cost

➢ If order quantity is small → ordering cost increases

➢ If order quantity is large → holding cost increases

➢ EOQ finds the quantity where total cost is minimum.

Assumptions:

1. Demand is constant and known
2. Lead time is fixed
3. Replenishment is instantaneous
4. No shortages are allowed
5. Ordering cost is constant
6. Holding cost is constant
7. Purchase price is constant

Formulas:

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Economic Order Quantity: $EOQ = \sqrt{\frac{2DS}{H}}$

Total Inventory Cost: $TC = \frac{D}{Q}S + \frac{Q}{2}H$

Reorder Point: $ROP = D \times L$

Problem:

The annual demand is 42,000 bottles.

Ordering cost per order is ₹750.

Holding cost is ₹18 per bottle per year.

A pharmaceutical distributor supplies a particular medicine to hospitals.

Additional information:

- Safety stock maintained = 400 bottles
- Lead time = 15 days
- Working days in a year = 350 days
- Purchase price per bottle = ₹120

Find:

1. EOQ
2. Number of orders per year
3. Reorder point including safety stock
4. Total inventory cost (excluding purchase cost)
5. Total annual cost including purchase cost
6. Average inventory including safety stock

Solution:

Given: D = 42,000, S = 750, H = 18, L = 15 days ,Safety stock = 400

1. EOQ:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(42000)(750)}{18}} = \sqrt{\frac{2(31500000)}{18}} = \sqrt{\frac{63000000}{18}}$$

$$= \sqrt{3500000} = 1870.82869338$$

$$EOQ \approx 1871\ bottles$$

2. Number of Orders: $N = \frac{D}{Q} = \frac{42000}{1871} = 22.447888 \approx 22.45\ orders.$

3. Reorder Point**:**

$$\text{Daily demand: } d = \frac{Annual\ Demand}{Working\ Days} = \frac{42000}{350}$$

$$= 120\ bottles.$$

$$ROP = (Dailydemand \times Leadtime) + Safetystock$$

$$= d \times L + SS = 120(15) + 40 = 1800 + 400$$

$$Reorder\ Point = 2200\ bottles.$$

4. Total Inventory Cost: $TC = \frac{D}{Q}S + \frac{Q}{2}H$

$$= \frac{42000}{1871} \times 750 + \frac{1871}{2} \times 18$$

$$= 16836 + 16839 = ₹33{,}675$$

5. Total Cost Including Purchase: $Total = D(d) + TC$

$$= 42000(120) + 33675$$

$$Total = ₹50{,}73{,}675.$$

6. Average Inventory: $I_{avg} = \frac{Q}{2} + SS$

$$= \frac{1871}{2} + 400 = 936 + 400$$

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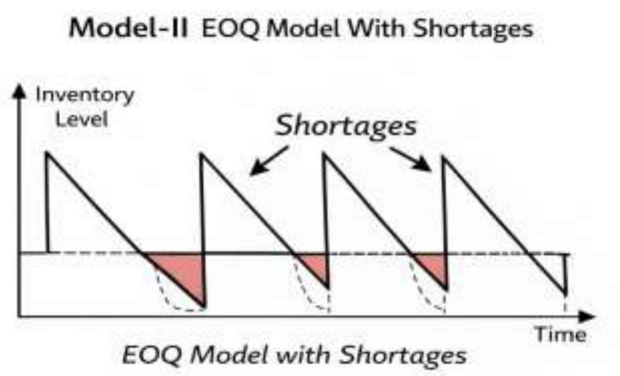
$$\therefore I_{avg} = 1336$$

MODEL II – EOQ Model with Shortages:

Definition:

EOQ model with shortages is an extension of the basic EOQ model where temporary shortages or backorders are allowed.



Concept:

In this model:

- Some demand is allowed to be backordered
- Shortage cost is introduced
- Optimal order quantity and maximum shortage level are determined

Assumptions:

1. Demand is constant
2. Shortages are permitted
3. Lead time is known
4. Costs are constant
5. Replenishment is instantaneous

Formulas:

Optimal Order Quantity: $Q = \sqrt{\frac{2DS(H+P)}{HP}}$

Maximum Shortage Level: $B = \frac{QH}{H+P}$

Total Cost: $TC = \frac{D}{Q}S + \frac{(Q-P)^2}{2Q}H + \frac{B^2}{2Q}P$

Problem:

A spare parts dealer experiences demand of 21,600 units per year.

Ordering cost = ₹500

Holding cost = ₹10 per unit per year

Shortage cost = ₹25 per unit per year

Additionally:

- Backorders are allowed
- Lead time = 8 days
- Working days = 360

Find:

1. Optimal order quantity
2. Maximum shortage
3. Maximum inventory level
4. Total annual cost
5. Reorder point

Solution:

Given: D = 21,600, S = 500, H = 10, P = 25

1. Optimal Q: $Q = \sqrt{\frac{2DS(H+P)}{HP}}$

$$= \sqrt{\frac{2(21600)(500)(35)}{250}} = \sqrt{\frac{756000000}{250}} = \sqrt{3024000}$$

$$Q = 1739\ units$$

2. Maximum Shortage: $B = \frac{QH}{H+P} = \frac{1739(10)}{35} = \frac{17390}{35} = 496.8571$

$$B \approx 497\ units$$

3. Maximum Inventory: $I_{max} = Q - B = 1739 - 497 = 1242$
4. . Total Cost: $TC = \frac{D}{Q}S + \frac{(Q-B)^2}{2Q}H + \frac{B^2}{2Q}P$

$$= \frac{21600}{1739} \times 500 + \frac{(1739-497)^2}{2(1739)} \times 10 + \frac{497^2}{2(1739)} \times 25$$

$$= \frac{21600}{1739} \times 500 + \frac{(1242)^2}{3478} \times 10 + \frac{247009}{3478} \times 25$$

$$= 6210.465785 + 4435.20414 + 1775.510351$$

$$TC = ₹12{,}421$$

5. Reorder Point:

Daily demand: $d = \frac{D}{Working\ days} = \frac{21600}{360}$

$$d = 60$$

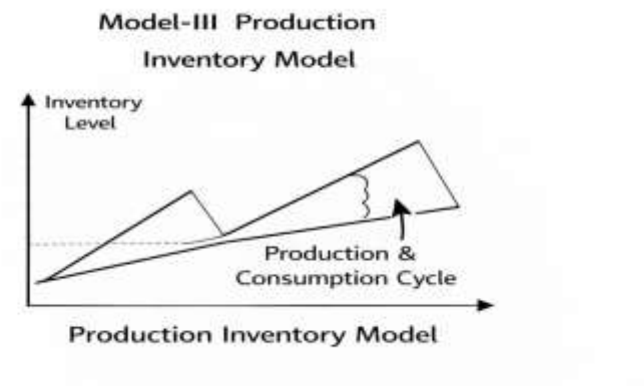
$$ROP = d \times Lead\ time = 60 \times 8$$

$$ROP = 480\ units$$

MODEL III – Production Inventory Model (EPQ Model):

Definition:

The Production Inventory Model (also called EPQ Model) is used when items are produced internally at a finite production rate rather than purchased.



Concept:

- Inventory builds gradually during production
- Demand and production occur simultaneously
- Production rate is greater than demand rate

This model determines the optimal production lot size.

Assumptions:

1. Production rate is finite
2. Production rate $P_r > D$
3. Demand is constant
4. No shortages allowed
5. Costs are known
6. Production and consumption occur simultaneously

Formulas:

Economic Production Quantity: $Q = \sqrt{\frac{2DS}{H} \cdot \frac{P_r}{P_r - D}}$

Maximum Inventory Level: $I_{max} = Q\left(1 - \frac{D}{P_r}\right)$

Total Cost: $TC = \frac{D}{Q}S + \frac{I_{max}}{2}H$

Problem:

A factory produces electronic meters internally.

Data:

- Annual demand = 16,000 units
- Production rate = 48,000 units per year
- Setup cost = ₹900 per run
- Holding cost = ₹14 per unit per year

Additional:

- Machine works 300 days a year
- Daily demand is uniform

Find:

1. EPQ
2. Maximum inventory
3. Total cost
4. Number of production runs
5. Length of production period
6. Idle time in each cycle

Solution:

1. EPQ: $Q = \sqrt{\frac{2DS}{H} \cdot \frac{P_r}{P_r - D}}$

$$= \sqrt{\frac{2(16000)(900)}{14} \cdot \frac{48000}{48000 - 16000}} = \sqrt{\frac{28800000}{14} \cdot \frac{48000}{32000}}$$

$$= \sqrt{2057142.857 \times 1.5} = \sqrt{3085714}$$

$$EPQ = 1757 \ units$$

2. Maximum Inventory: $I_{max} = Q\left(1 - \frac{D}{P_r}\right)$

$$= 1757\left(1 - \frac{16000}{48000}\right)$$

$$= 1757(1 - 0.333333333) = 1757(0.66666667)$$

$$I_{max} = 1171\ units$$

3. Total Cost: $TC = \frac{D}{Q}S + \frac{I_{max}}{2}H = \frac{16000}{1757} \times 900 + \frac{1171}{2} \times 14 =$ $8195.78 + 8197 \approx ₹16391$

4. Number of Runs: $N = \frac{D}{Q} = \frac{16000}{1757} = 9.1$

$$N \approx 9$$

5. Production Time: $t = \frac{Q}{P_r} = \frac{1757}{48000} = 0.0366\ years$

6. Idle Time:

   Cycle time: $T = \frac{Q}{D} = \frac{1757}{16000} = 0.1098$

   Idle time $= T - t = 0.1098 - 0.0366 = 0.0732\ year$

MODEL IV – EOQ Model with Quantity Discounts:

Definition:

This model considers situations where suppliers offer price discounts for ordering in larger quantities.

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Concept:

The objective is to select the order quantity that gives minimum total cost, considering:

- Purchase cost
- Ordering cost
- Holding cost

Assumptions:

1. Demand is known and constant
2. Price varies with order size
3. Replenishment is instantaneous
4. No shortages allowed
5. Discount structure is known

Formulas:

EOQ (ignoring discount): $EOQ = \sqrt{\frac{2DS}{H}}$

Total Cost Including Purchase Cost: $TC = DP + \frac{D}{Q}S + \frac{Q}{2}H$

Solution Procedure:

1. Compute EOQ

2. Check price break quantities
3. Compute total cost at EOQ and discount levels
4. Choose quantity with lowest total cost

Problem:

Annual demand = 36,000 units

Ordering cost = ₹600

Holding cost rate = 20% of price

Price schedule:

Quantity	Price
0-1999	₹50
2000-3999	₹47
4000+	₹44

Find optimal order quantity considering holding cost depends on price.

Solution:

For each level:

Holding cost: $H = 0.2 \times Price$

EOQ at price ₹50: $H = 0.2 \times 50 = ₹10$

$$H = 10$$

$$EOQ = Q_1 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(36000)(600)}{10}} = \sqrt{\frac{43200000}{10}}$$

$$= \sqrt{4320000}$$

$$Q_1 = 2078 units (approx)$$

Feasibility Check

This EOQ must lie in range 0-1999

But,

2078>1999; Not feasible (not in range)

So, we take boundary value: $Q_1 = 1999$

Total Cost for Feasible Quantity: $TC = DP + \frac{D}{Q}S + \frac{Q}{2}H$

$$Q = 1999, P = 50, H = 10, D = 36000, S = 600$$

$$TC_1 = 36000(50) + \frac{36000}{1999}(600) + \frac{1999}{2}(10)$$
$$= 1800000 + 10805.4 + 9995$$

$$TC_1 = ₹1,820,800.4$$

EOQ at price ₹47: $H = 0.2 \times 47 = ₹9.4$

$$EOQ = Q_2 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(36000)(600)}{9.4}} = \sqrt{\frac{43200000}{9.4}} = \sqrt{4595744.68}$$

$$Q_2 = 2144\, units(approx)$$

Feasibility Check:

This EOQ must lie in range =2000-3999

2144∈[2000,3999]

It is feasible

So, $Q_2 = 2144$

Total Cost for Feasible Quantity: $TC = DP + \frac{D}{Q}S + \frac{Q}{2}H$

$$Q = 2144, P = 47, H = 9.4, D = 36000, S = 600$$

$$TC_2 = 36000(47) + \frac{36000}{2144}(600) + \frac{2144}{2}(9.4)$$
$$= 1692000 + 10075.2 + 10076.8$$

$$TC_2 = ₹1,712,152$$

EOQ at price ₹44: $H = 0.2 \times 44 = ₹8.8$

$$H = 8.8$$

$$EOQ = Q_3 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(36000)(600)}{8.8}} = \sqrt{\frac{43200000}{8.8}} = \sqrt{4909090.91}$$

$$Q_3 = 2216\, units(approx)$$

Feasibility Check

This EOQ must lie in range=4000 and above

2216<4000. Not feasible (not in range)

So, we take minimum of range:$Q_3 = 4000$

Total Cost for Feasible Quantity: $TC = DP + \frac{D}{Q}S + \frac{Q}{2}H$

$$Q = 4000, P = 44, H = 8.8, D = 36000, S = 600$$

$$TC_3 = 36000(44) + \frac{36000}{4000}(600) + \frac{4000}{2}(8.8)$$
$$= 1584000 + 5400 + 17600$$

$$TC_3 = ₹1,607,000$$

Compare Total Costs:

Level	Order Quantity	Total Costs
1	1999	₹18,20,800.4
2	2144	₹17,12,152
3	4000	₹16,07,000

∴Minimum Total Cost=₹16,07,000

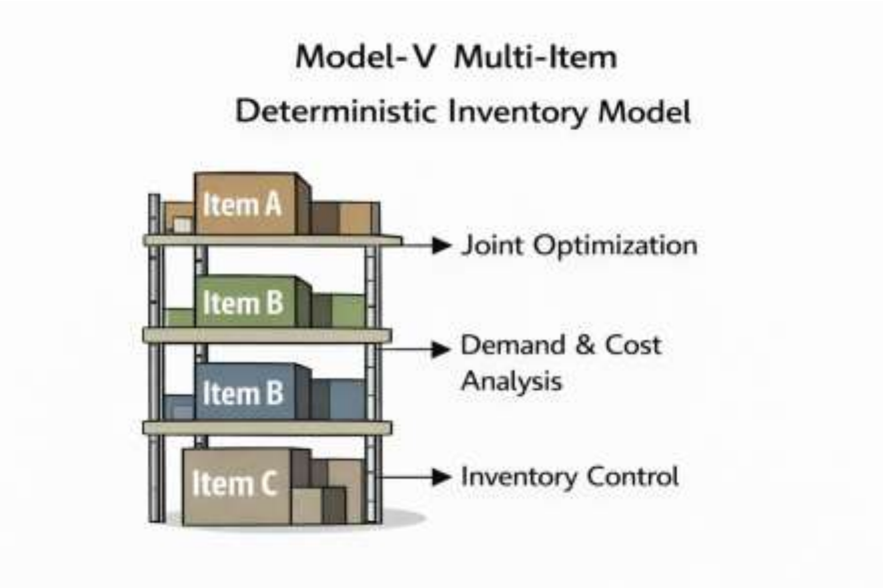
Optimal Order Quantity is 4000 units

The optimal order quantity considering price-depending holding cost is: $400\ units\ at\ price$ ₹44

MODEL V – Multi-Item Deterministic Inventory Model:

Definition:

The Multi-Item Inventory Model deals with more than one item simultaneously under certain constraints like limited budget or storage space.



Concept:

In real life:

- Organizations manage many items
- Resources are limited

Therefore, order quantities must be optimized jointly instead of individually.

Assumptions:

1. Demand for each item is known
2. Costs are known
3. More than one item is considered
4. There exists a constraint:
   - Budget, or
   - Storage space

Formulas:

EOQ for each item (without constraint): $Q_i = \sqrt{\frac{2D_iS_i}{H_i}}$

Objective Function: $Minimize TC = \sum_{i=1}^{n}\left(\frac{D_i}{Q_i}S_i + \frac{Q_i}{2}H_i\right)$

Constraint Example (Space Constraint): $\sum_{i=1}^{n} a_iQ_i \leq A$

Problem:

A company manages three products.

Item	Demand D_i	$Ordering\ Cost\ S_i$	$Holding\ Cost\ H_i$	Price P_i
A	9000	300	12	40
B	6000	250	10	60
C	4000	200	8	50

Total available purchasing budget per order cycle = ₹2,00,000

Find:

1. Individual EOQs
2. Check budget feasibility
3. Adjust order quantities to satisfy constraint

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Solution:

EOQ for each item (without constraint): $Q_i = \sqrt{\frac{2D_iS_i}{H_i}}$

For Item A: $Q_A = \sqrt{\frac{2D_1S_1}{H_1}}$

We Have, $D_1 = 9000 , S_1 = 300 , H_1 = 12$

$$Q_A = \sqrt{\frac{2(9000)(300)}{12}} = \sqrt{\frac{5400000}{12}} = \sqrt{450000}$$

$$Q_A = 671 \; units$$

For Item B: $Q_B = \sqrt{\frac{2D_2S_2}{H_2}}$

We Have, $D_2 = 6000 , S_2 = 250 , H_2 = 10$

$$Q_B = \sqrt{\frac{2(6000)(250)}{10}} = \sqrt{\frac{3000000}{10}} = \sqrt{300000}$$

$$Q_B = 548$$

For Item C: $Q_C = \sqrt{\frac{2D_3S_3}{H_3}}$

We Have, $D_3 = 4000 , S_3 = 200 , H_3 = 8$

$$Q_C = \sqrt{\frac{2(4000)(200)}{8}} = \sqrt{\frac{1600000}{8}} = \sqrt{200000}$$

$$Q_C = 447$$

Budget Check:

$$P_1 = 40, P_2 = 60, = P_3 = 50$$

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$$P_1Q_A + P_2Q_B + P_3Q_C = 40(671) + 60(548) + 50(447)$$
$$= 26840 + 32880 + 22350 = ₹82070$$

Budget Analysis:

- Required purchasing budget=₹82,070
- Available budget=₹2,00,000
- Budget Constraint: Satisfied

Since within budget→EOQs are feasible

Final Optimal Quantities is:

- A = 671 units
- B = 548 units
- C = 447 units

CONCLUSION:

Deterministic Inventory Control Models provide a systematic method for determining optimal order quantity and minimizing total inventory cost. Models such as EOQ, EOQ with shortages, Production Inventory model, and Quantity Discount model help improve efficiency and cost control in inventory management. Although these models assume constant demand and known parameters, they form the basic foundation of inventory theory. While probabilistic models are useful for uncertain conditions, deterministic models remain important tools for effective planning and decision-making**.**

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